Homework #1 Solutions

Section 5.5

12. a) $\int \frac{dx}{\sqrt{5x+8}}$

$u = 5x+8$

$du = 5\,dx$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{1}{5}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{5}\int u^{-1/2}\,du$$

$$= \frac{1}{5}\cdot 2u^{1/2} + C$$

$$= \boxed{\frac{2}{5}\sqrt{5x+8} + C}$$

b) $u = \sqrt{5x+8}$

$u^2 = 5x+8$

$2u\,du = 5\,dx$

$$\int \frac{dx}{\sqrt{5x+8}} = \frac{2}{5}\int \frac{u}{u}\,du$$

$$= \frac{2}{5}\int 1\,du = \frac{2}{5}u + C = \boxed{\frac{2}{5}\sqrt{5x+8} + C}$$

16. $\int \frac{3\,dx}{(2-x)^2}$

$u = 2-x$

$du = -dx$

$$\int \frac{3\,dx}{(2-x)^2} = -3\int u^{-2}\,du = (-3)(-1)u^{-1} + C$$

$$= \boxed{\frac{3}{2-x} + C}$$

30. $\int \frac{6\cos t}{(2+\sin t)^3}\,dt$

$u = 2+\sin t \qquad du = \cos t\,dt$

$$\int \frac{6\cos t}{(2+\sin t)^3}dt = 6\int u^{-3}\,du = \frac{6\cdot u^{-2}}{-2} + C$$

$$= \boxed{\frac{-3}{(2+\sin t)^2} + C}$$

44. $\int \frac{\ln\sqrt{t}}{t}\,dt$

$u = \ln\sqrt{t} \qquad du = \frac{1}{\sqrt{t}} \cdot \frac{1}{2}\frac{1}{\sqrt{t}}\,dt = \frac{1}{2t}\,dt$

$\int \frac{\ln\sqrt{t}}{t}\,dt = 2\int u\,du = u^2 + C = \boxed{(\ln\sqrt{t})^2 + C}$

56. a) $\int \sqrt{1+\sin^2(x-1)} \cdot \sin(x-1)\cos(x-1)\,dx$

$u = x-1 \qquad du = dx$

$\int \sqrt{1+\sin^2 u}\,\sin u \cos u\,du$

$v = \sin u \qquad dv = \cos u\,du$

$\int \sqrt{1+v^2} \cdot v\,dv$

$w = 1+v^2 \qquad dw = 2v\,dv$

$\frac{1}{2}\int \sqrt{w}\,dw = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} w^{3/2} + C = \frac{1}{3}(1+v^2)^{3/2} + C$

$= \frac{1}{3}(1+\sin^2 u)^{3/2} + C = \boxed{\frac{1}{3}(1+\sin^2(x-1))^{3/2} + C}$

b). $u = \sin(x-1) \qquad du = \cos(x-1)\,dx.$

$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \int \sqrt{1+u^2} \cdot u\,du$

$v = 1+u^2 \qquad dv = 2u\,du$

$\frac{1}{2}\int \sqrt{v}\,dv = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} v^{3/2} + C = \frac{1}{3}(1+u^2)^{3/2} + C$

$= \boxed{\frac{1}{3}(1+\sin^2(x-1))^{3/2} + C}$

c) $u = 1+\sin^2(x-1) \qquad du = 2\sin(x-1) \cdot \cos(x-1)\,dx.$

$\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx = \frac{1}{2}\int \sqrt{u}\,du$

$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = \boxed{\frac{1}{3}(1+\sin^2(x-1))^{3/2} + C}$

4. $\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x \, dx$

$du = 2x\,dx \qquad v = -\cos x$

$$\int x^2 \sin x \, dx = -x^2 \cos x + \int 2x \cos x \, dx$$

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad V = \sin x$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2\left[x \sin x - \int \sin x \, dx\right]$$

$$= \boxed{-x^2 \cos x + 2x \sin x + 2\cos x + C}$$

7. $\int \tan^{-1} y \, dy$

$u = \tan^{-1} y \qquad dv = dy$

$du = \frac{1}{1+y^2}\,dy \qquad v = y$

$$\int \tan^{-1} y \, dy = y \tan^{-1} y - \int \frac{y}{1+y^2}\,dy$$

$u = 1+y^2 \qquad du = 2y\,dy$

$$\int \tan^{-1} y \, dy = y \tan^{-1} y - \frac{1}{2}\int \frac{1}{u}\,du$$

$$= y \tan^{-1} y - \frac{1}{2}\ln|u| + C$$

$$= \boxed{y \tan^{-1} y - \frac{1}{2}\ln(1+y^2) + C}$$

22. $\int e^{-y} \cos y \, dy$

$u = e^{-y}$ $\quad dv = \cos y \, dy$

$du = -e^{-y} \, dy$ $\quad v = \sin y$

$$\int e^{-y} \cos y \, dy = e^{-y} \sin y + \int e^{-y} \sin y \, dy$$

$u = e^{-y}$ $\quad dv = \sin y \, dy$

$du = -e^{-y} \, dy$ $\quad v = -\cos y$.

$$\int e^{-y} \cos y \, dy = e^{-y} \sin y + -e^{-y} \cos y - \int e^{-y} \cos y \, dy$$

So $2 \int e^{-y} \cos y \, dy = e^{-y} \sin y - e^{-y} \cos y$

and $\boxed{\int e^{-y} \cos y \, dy = \frac{e^{-y} \sin y - e^{-y} \cos y}{2}}$

26. $\int_0^1 x \sqrt{1-x} \, dx$

$u = 1-x$ $\quad du = -dx$.

lower bound: $u = 1-0 = 1$
upper bound: $u = 1-1 = 0$

$$-\int_1^0 (1-u)\sqrt{u} \, du = \int_1^0 (u-1)\sqrt{u} \, du$$

$v = u-1$ $\quad dw = \sqrt{u} \, du$

$dv = du$ $\quad w = \frac{2}{3} u^{3/2}$

$$\int_1^0 (u-1)\sqrt{u} \, du = \frac{2}{3} u^{3/2}(u-1) \Big|_1^0 - \int_1^0 \frac{2}{3} u^{3/2} \, du$$

$$= \left(\frac{2}{3} u^{5/2} - \frac{2}{3} u^{3/2}\right) \Big|_1^0 - \frac{2}{3} \cdot \frac{2}{5} u^{5/2} \Big|_1^0$$

$$= -\frac{2}{3} + \frac{2}{3} - \left(0 - \frac{4}{15}\right)$$

$$= \boxed{\frac{4}{15}}$$

40. Show: $\int x^n \sin x \, dx = -x^n \cos x + n \int x^{n-1} \cos x \, dx$.

$u = x^n$ $\quad dV = \sin x \, dx$

$du = n x^{n-1} dx$ $\quad V = -\cos x$.

So $\int x^n \sin x \, dx = -x^n \cos x + \int n x^{n-1} \cos x \, dx$ ✓